MATHEMATICS EDUCATION OF TOMORROW

In January 2004, Professor Bozenna Pasik-Duncan received the AWM Louise Hay Award for Contributions to the Mathematics Education for her broad and inspiring vision of mathematics as a discipline and as a profession, and for her remarkable skill and commitment in carrying out the role of a professional mathematician in a wide variety of communities and settings. For a bio of Pasik-Duncan, see the March–April 2004 issue of this newsletter or www.awm-math.org/hayaward/2004.html. The text of her lecture given on August 14 at the 2004 MathFest in Providence, RI follows.

I would like to thank Carolyn Gordon, the President of the Association for Women in Mathematics for her kind introduction.

This lecture is delivered in the memory of Louise Hay—teacher, scholar, administrator and human being. Moreover, it is dedicated to scholars who teach introductory mathematics courses, in particular, to those scholars who teach honors calculus courses.

Through this talk, I hope you will learn that what I call Mathematics Education of Tomorrow is a collaborative effort integrating scholarship, teaching and learning. This collaborative effort needs to include K–12 teachers, students and scholars. We need to work together as partners who are all learners in the process of math education.

At this MathFest meeting and at many talks that I have attended, I have learned that many of you have important roles in the teaching and learning process. What I want to talk about today is how I put all of these pieces together in one controlled system that has been working well in my Stochastic Adaptive Control Group at KU. I call it a controlled system and I, as a scholar and teacher, will play the role of controller.

Since we need to know the controller that we will be using in our system, let me start by describing my education and research background. I am proud of being broadly educated; my academic background ranges from Numerical Analysis through Stochastic Processes to Stochastic Adaptive Control and includes a formal pedagogical training component. My research passion for the past twenty years or so has been adaptive control of stochastic linear and nonlinear systems. The general approach to adaptive control exhibits a splitting, or separation, of identification and adaptive control. Industrial models can be often described as controlled systems. The system's behavior depends on the parameters, and the fact that the values of the parameters are unknown makes the system unknown. Some crucial information concerning the system is not available to the controller, and this information should be learned during the system's performance. The described problem is the basic problem of adaptive control. The system can be described by a stochastic differential equation or a partial differential equation. The solution to the adaptive control problem consists of showing the strong consistency of the family of estimators of an unknown parameter and the self-optimality of an adaptive control that uses the family of estimates. The disturbance or what we call a noise in the system is modeled by a Brownian motion or more generally by a fractional Brownian motion (more accurate for recent problems in telecommunication, finance or biomedicine).

The beauty of this area of research lies in its use of many different areas of mathematics: from functional analysis through stochastic processes, stochastic analysis, stochastic calculus of a fractional Brownian motion, stochastic partial differential equations, stochastic optimal control to methods of mathematical statistics, as well as current computational methods in stochastic differential and partial differential equations. Let me describe the concept of an adaptive control using an investment model with transaction fees. Consider a model where an investor has free choice in investing in two assets, a bond with a fixed rate of growth or a stock whose growth is governed by a Brownian motion with unknown drift and variance. The investor controls his asset by transferring money between the stock and the bond. The control variable is the total amount of money transferred from the stock to the bond. The bond is governed by the differential equation and the stock is governed by a stochastic differential equation. The goal is to find the optimal control so that the expected rate of growth is maximized. The identification problem is to estimate the unknown parameters, the drift and variance, based on the available observations. The adaptive control problem is to construct the (certainty equivalence) adaptive control as a function of the state and the current estimate.

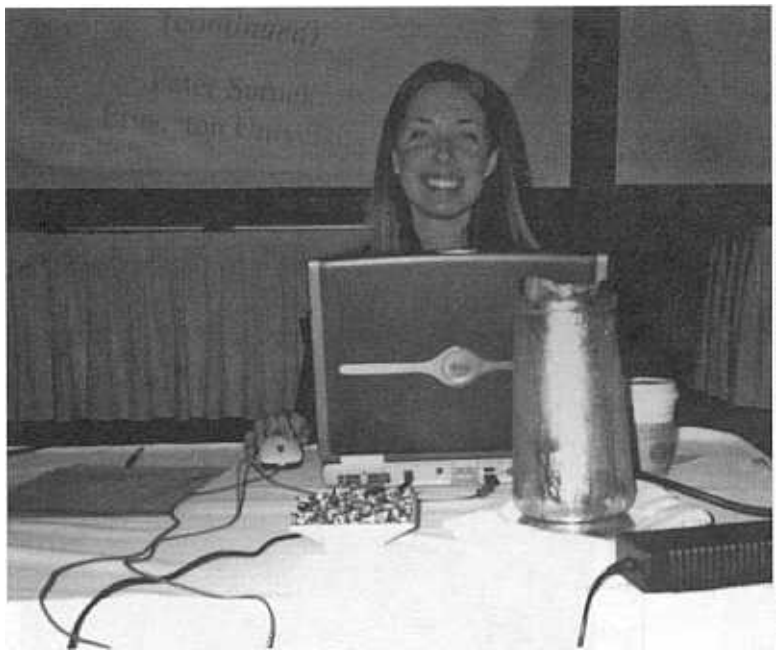
Let me describe now our teaching and learning process. It is both vertical and horizontal. Vertical integration incorporates students and researchers at different levels in their teaching and learning approach. It involves and engages K–12, undergraduate, graduate and postdoctoral students. Horizontal integration incorporates faculty from various disciplines in the teaching of a course. The classroom forms a system. It is stochastic because there is a lot of randomness in the classroom. Students talk, fall asleep or bring a baby to the classroom (as happened recently in one of my calculus classes for non-math-major students). What would you do? I cannot very well tell the mother to take her baby away. As a controller I make a quick decision. I decided to teach the baby, who was roaming around on the carpet in front of me while his mother was focused on learning and taking notes from the blackboard. I believe that the baby will be a great mathematician. At the age of seven months he learned some calculus! Having that baby in my class has become symbolic for my teaching. I enjoy explaining math concepts or my research to people who are not mathematicians or who have never even learned mathematics. I teach students to do the same.

In systems theory, we analyze every system carefully. We analyze the existence of a solution and computational aspects of it, we simulate stochastic equations, we collect information, we compose results, etc. We do the same in our teaching. I like to treat a classroom as my scientific laboratory. I collect information. As I introduce myself, I ask the students to introduce themselves. We as teachers need to know the students and their interests. This information is important when we design projects for them. There are many unknowns in this system, so we need to estimate (learn) them as in the theory, and at each instant a controller/teacher uses this new estimate in control strategies and adapts the system. In theory this is called adaptation. It is the same in teaching. We collect information, we build a portfolio, we analyze our reports and data after every class, and we also want to do better each time so that in the long run we will do as well as if we knew the system perfectly. In the theory of stochastic adaptive control, this property of adaptive control is called self-tuning.

As you can see this method of teaching is called scholarship in teaching. We treat teaching as a stochastic process that changes over time, a process with several components such as vision, design, data collection and data analysis. We integrate teaching and learning. As in the theory, the controller has to learn, so a teacher, as a controller, has to learn too, and the system has to learn, meaning the students have to learn. In control theory, we love feedback control. In teaching, we love feedback from students. For example, a student who graduated last May who got her "dream job" at the Mayo clinic says:

Bozenna Pasik-Duncan with Wendell Fleming (Brown University) after her talk

> My job is going well. I get to work on research projects that couldn't be done at most hospitals. Mayo is at the forefront of new technologies and techniques, the doctors see many patients with rare diseases to study, and they have extensive medical records to work from that go back to the early 1900's! At least once a week, I get to meet with an investigator to discuss their research or results that I have obtained. Working is a big change from school, since now the sets of data are much larger and much more complicated than anything you'd find in a textbook. I have learned a lot in just under two months. I'd like to mention that I'm glad that you had people from other departments come and speak with us during seminar and AWM meetings. When I meet with investigators, they don't talk about math. They talk about genetics, anesthesiology, surgery, cancer, technology, etc. It's important to be able to understand other subjects so that you can thoughtfully apply mathematics to them.



Dominique Duncan, University of Chicago, helping her mother at the talk

We learn from this feedback that collaborative efforts in research and collaborative efforts in teaching are important. Communication and writing are equally important, so we need to be sure that we teach students how to communicate and how to write. We need to be sure that our graders will grade students' papers the way we review papers for professional journals. We make comments and corrections with the highest respect for the authors, and we expect our graders to do the same. This isn't the place for comments like "what is this?" It is important to engage students in the voyage of learning. I like the following quote of Marcel Proust: "The voyage of discovery lies not in finding new landscapes, but in having new eyes."

It is rewarding to bring someone who works in probability to your calculus class when you give an example of an integral as the expected value of a random variable. Students like visitors, and a probabilist will give them real world problems and will relate them to his own research. I use the language of my research freely. When I teach calculus, I make connections to stochastic calculus, an area of my research. Making connections has been fascinating. When I talk about curvature in calculus or if I talk about random processes in stochastic analysis, I bring my research collaborators from the KU Medical Center, from Flint Hills Scientific LLC, KU Business School and Economics Department, from KU Information and Telecommunication Technology Center, from Sprint Corporation or from the Music Department. Collaborators from the medical center talk about epilepsy and seizures, and they show brain waves and explain how you can use a curvature concept to predict seizures. However, because most people have never experienced an epileptic seizure, they have no feel for these graphs. The musicians can modify a Mozart sonata by adding a noise in the orchestra, duplicating the frequency behavior in powerful ways. If you bring your research collaborators, especially those from other countries, to your classroom with good preparation for their visits, this changes students' perspectives. It opens their eyes and it wakes up their imagination and creativity. They see how math can be found everywhere and how often it is hidden. They see the role of the broad and inspiring vision of mathematics. They see the power, beauty and excitement of the cross-boundary nature of math, and this is what they write in their reflections from the classroom:

> Taking this class really opened my eyes to math. Instead of dreading and avoiding math, I love it. She teaches until we know what is going on and not to follow a schedule.... She taught us statistics with a true passion for the subject. She taught us how practical it is in life and how relevant it is for any field of study we choose.... I learned from the material, learned from myself, and for it all, my life is better. A course with that kind of an impact and outcome is definitely life changing.

By sharing with students your research collaborators, in particular, those from other countries, other cultures, we show what roles professional mathematicians play in a wide variety of communities and settings. We organize Workshops on Control for High School Teachers of Math and Science and Students at all major control conferences. They are sponsored by the NSF and the Control Systems Society, American Automatic Control Council and International Federation of Automatic Control. We organized them in Chicago, Denver, Las Vegas and Maui. The number of participants grows exponentially. The last one was held for 300 students. The

request was for 1,000. Next year will be very busy with workshops in Oregon, Cyprus, Czech Republic, Spain and later in California and New York. The speakers are outstanding national and international control researchers who are also outstanding teachers. They are excited about sharing their love for research, about showing middle and high school students that control is everywhere, and they pass the message that to know and understand control well they need to know math well.

We go filled with our excitement and love for math to local schools; we invite sixth graders to the university for workshops run by our undergraduates, graduates and faculty. The University of Kansas Math Department has become a friendly place in our community. Our AWM Student Chapter builds important bridges between the KU Math Department and local schools. We build other bridges: with the Engineering and Business Schools, with the Medical School and Psychology Department as well as with the departments of economics and music. We travel everywhere around the world to visit our collaborators, and we talk with them about their research and teaching experiences, but we also send our students, including undergraduates, to international conferences and to meet students there. This has been terrific—the most successful practice. I don't need to be stressed out by having an accent. I am proud of knowing several languages. I teach Polish in all my classes as a nice break from difficult mathematical concepts. KU students travel to Poland, and as a result, I couldn't find a seat on any plane to Poland last July! I developed the reputation "with her as a teacher, math is not as difficult as that Polish...."

Our KU Stochastic Adaptive Control group of students has been very successful in getting jobs in industry ranging from financial companies and international banks through actuarial and telecommunication companies to biomedical research institutions. They are good citizens, and they are important contributors to real world problem solving. They are involved in interdisciplinary research. They serve as important advisors to KU students: "Always be alert in your analysis of what you are doing—ask yourselves why things are the way they are—see if you can explain what you observe. Make sure that you consider how even the most abstract mathematics you do can be applied to real life problems."

Let me finish my talk by challenging you. Since this talk is sponsored by AWM, and since my daughter just

NSF-AWM TRAVEL GRANTS FOR WOMEN

The objective of the NSF-AWM Travel Grants program is to enable women to attend research conferences in their fields, thereby providing a valuable opportunity to advance their research activities and their visibility in the research community. By having more women attend such meetings, we also increase the size of the pool from which speakers at subsequent meetings may be drawn and thus address the persistent problem of the absence of women speakers at some research conferences.

Travel Grants. These grants provide full or partial support for travel and subsistence for a meeting or conference in the applicant's field of specialization. A maximum of $1000 for domestic travel and of $2000 for foreign travel will be applied. For foreign travel, US air carriers must be used (exceptions only per federal grants regulations; prior AWM approval required).

Eligibility. These travel funds are provided by the Division of Mathematical Sciences of NSF, and the research conference must be in an area supported by DMS. (See http://www.nsf.gov/od/lpa/news/publicat/nsf03009/mps/dms.htm#1 for the list of supported areas.) Applicants must be women holding a doctorate (or equivalent experience) and having a work address in the US (or home address, in the case of unemployed mathematicians). Anyone who has been awarded an AWM-NSF travel grant in the past two years is ineligible. Anyone receiving significant external governmental funding (more than $1000 yearly) for travel is ineligible. Partial travel support from the applicant's institution or from a non-governmental agency does not, however, make the applicant ineligible.

Target dates. There are three award periods per year. An applicant should send *five* copies of 1) a cover letter, including the conference name, conference dates and location (city/state/country), and amount of support requested, 2) a description of her current research and of how the proposed travel would benefit her research program, 3) her curriculum vitae, 4) a budget for the proposed travel, and 5) a list of all current and pending travel funding (governmental and non-governmental) and the amounts available for your proposed trip to: Travel Grant Selection Committee, Association for Women in Mathematics, 4114 Computer & Space Sciences Building, University of Maryland, College Park, MD 20742-2461. If you have questions, contact AWM by phone (301-405-7892) or email (awm@math.umd.edu). Applications via email or fax will not be accepted. The next two deadlines for receipt of applications are **February 1** and **May 1, 2005**. *Please note that funds are pending for the May 1, 2005 cycle.*

finished the first year of the undergraduate math program, I ask each and all of you to encourage first year female students to stay in math by showing them that math will take them everywhere. As I always say, they can be better lawyers if they know math well. I consider losing a female student in a first year honors calculus or in any honors calculus as a personal failure.

Thank you for giving me the opportunity to share with you my passion, enthusiasm and love for teaching, learning and doing research. Teaching, learning, collaborating, and making connections with probability and stochastic modeling make me the happiest person on this planet.

Big thanks go to: AWM, MAA, KU Stochastic Adaptive Control Group and KU Students, NSF, KU Mathematics Department, Control Systems Society, my collaborators: KU Medical Center Flint Hills Scientific LLC, KU Business School, and KU Information and Telecommunication Technology Center. Special thanks go to: Dominique Duncan, University of Chicago and Tyrone Duncan, KU Math Department; Mary Klyder, KU Honors Program; Daniel Bernstein, KU Center for Teaching Excellence; and Mary Jane Dunlap, KU University Relations.

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COLLABORATIVE RESEARCH GRANTS FOR WOMEN

Dedicated to the memory of Ruth Michler

AWM will continue to offer Collaborative Research Grants to enable women who are already tenured to carry out collaborative research at other institutions. (Women who are not yet tenured are referred to the Mentoring Grants Program.) We anticipate offering one or two grants for amounts up to $2500 in 2005. Each grant may be used to fund travel, accommodations, and other required expenses for a tenured woman mathematician to travel to an institute or a department to do research with a specified individual. All travel must be completed within one year of the award. For foreign travel, US air carriers must be used (exceptions only by prior approval from AWM).

Applications: Applicants must be women holding tenure or equivalent experience and must have a work address in the US. The applicant's research must be in a field that is supported by the Division of Mathematical Sciences of the National Science Foundation. (See http://www.nsf.gov/od/lpa/news/publicat/ nsf03009/mps/dms.htm#1 for the list of supported areas.)

An application should consist of: 1) a cover letter; 2) a curriculum vita; 3) a research proposal (approximately five pages in length) which specifies why the proposed travel would be particularly beneficial; 4) a supporting letter from the proposed collaborator (who must indicate his/her availability at the proposed travel time), together with the curriculum vita of the proposed collaborator; 5) a proposed budget; and 6) information about other sources of funding available to the applicant. A final report will be required from each awardee. Awards will be determined on a competitive basis by a selection panel consisting of distinguished mathematicians appointed by the AWM.

Send *five* complete copies of the application materials (including the cover letter) to: Collaborative Research Grant Selection Committee, AWM, 4114 Computer & Space Sciences Building, University of Maryland, College Park, Maryland 20742-2461. For further information: phone 301-405-7892, email awm@math.umd.edu, or visit www.awm-math.org. Applications must be received by **February 1, 2005**; applications via email or fax will not be accepted.